LARGE SAMPLE AND JACKKNIFE PROCEDURES FOR SMALL SAMPLE ORTHOGONAL LEAST SQUARES INFERENCE

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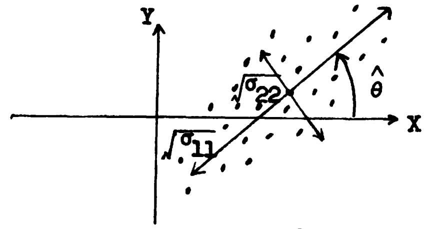
Exact distribution theory for orthogonal least-squares estimators is quite difficult (this is a special case of principal components estimation). In general the estimators have asymptotic normal distributions. Using this fact and using a consistent estimator for the variance, approximate confidence intervals can be obtained. An alternative is to use the jackknife technique to obtain confidence intervals. A Monte Carlo study shows that even for small samples the two procedures tend to give similar results.

$$\hat{\theta} = \frac{1}{2}\tan^{-1}\frac{2A_{12}}{A_{11}-A_{22}}$$

$$A_{11} = \sum_1^N (X_i - \bar{X})^2,\quad A_{12} = \sum_1^N (X_i-\bar{X})(Y_i-\bar{Y}),$$

$$A_{22} = \sum_1^N (Y_i - \bar{Y})^2$$



slope = tan $\hat{\theta}$

$\hat{\theta}$ asymptotically $N(\theta, \sigma^2_{\theta,N})$. X,Y bivariate normal implies

$$\sigma^2_{\theta,N} = \frac{1}{N}\cdot\frac{\sigma_{11}\sigma_{22}}{(\sigma_{11}-\sigma_{22})^2} = \frac{1}{N}\cdot\frac{\sigma_{11}/\sigma_{22}}{(\sigma_{11}/\sigma_{22}-1)^2}$$

Large Sample Confidence Interval: $\hat{\theta} \pm z_\alpha \cdot \hat{\sigma}_{\theta,N}$

Jackknife Confidence Interval: $\tilde{\theta} \pm t_{\alpha,K-1}\cdot s_\theta/\sqrt{K}$

where $\tilde{\theta} = \frac{1}{K}\sum_1^K \tilde{\theta}_i$, $s^2_\theta = \frac{1}{K-1}\sum_1^K(\tilde{\theta}_i - \tilde{\theta})^2$

Numbers of 90% Confidence Intervals for θ which do not contain the true value.

Normal - Normal		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	13	11	11	9
30	10	13	11	11	14
10	10	27**	21**	24**	11

Uniform Uniform		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	10	10	10	9
30	10	10	13	10	11
10	10	19**	11	16*	8

Uniform Exponential		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	13	10	11	9
30	10	13	11	11	9
10	10	19**	10	16*	7

*Significantly different from 10 at 5% level
**Significantly different from 10 at 1% level

Numbers of 95% Confidence Intervals for θ which do not contain the true value.

Normal - Normal		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	7	3	5	4
30	10	7	7	5	6
10	10	20**	13**	11**	6

Uniform Uniform		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	9	9	9	9
30	10	9	9	9	8
10	10	12**	7	9	4

Uniform Exponential		σ_{11}/σ_{22} = 4		σ_{11}/σ_{22} = 100	
N	K	Lg.Sample	Jack-knife	Lg.Sample	Jack-knife
30	30	9	5	6	5
30	10	9	6	6	6
10	10	13**	4	11**	4

*Significantly different from 5 at 5% level
**Significantly different from 5 at 1% level